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# Optimal carbon policies in a dynamic heterogeneous world[☆]

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ABSTRACT

We derive the optimal contributions to global climate policy when countries differ with respect to income level and pollution intensity. Countries' growth rates are determined endogenously, and abatement efficiency is improved by technical progress. We show that country heterogeneity has a crucial impact on optimal policy contributions: more developed countries have to make a larger effort while less developed countries are allowed to graduate under a less stringent environmental regime. The optimal allocation of pollution permits depends on international trade. In the absence of international permit trade, more developed countries should receive more permits than the less developed countries, but permit prices are higher in the rich countries and eventually countries converge in income. With international permit trade, more developed countries receive less permits than the less developed. When global distribution of physical capital is uneven and the aggregate pollution ceiling is low, poor countries receive all the permits.

## 1. Introduction

How much should each country contribute to global climate policy? This is a consequential question in international climate negotiations but less a focus of standard economics. The main contribution of this paper is to derive the determinants of policy contributions that are both optimal and equitable from the perspective of global welfare. Using a fully dynamic model, we determine an optimal burden sharing in international climate policy that takes into account the heterogeneity of countries, especially with respect to their development. We show why and how more developed countries need to contribute more to international climate policy. In the case where pollution permits cannot be traded internationally, we find that in the optimum prices for pollution permits are higher in developed countries. In fully integrated international markets for pollution rights, more developed countries should initially receive fewer permits, or even none at all, if the global capital stock is very unevenly distributed.

Up to now economists have been primarily concerned with efficiency and the objective of reaching internationally agreed temperature targets at minimal cost (Cramton et al., 2015). However, a policy assessment includes distributional issues, in particular in a world which is very heterogeneous. Indeed, international climate negotiations have revealed significant differences in countries' negotiation positions, which are often related to the stage of development and the carbon intensity of the economy. Equity and fairness are prime concerns of climate negotiators and policy makers: the distribution of the burden of a global policy is central to them and their electorate. The implementation of stringent policies and further progress in international climate negotiations will thus depend on whether country contributions are perceived as both equitable and efficient.

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It is known that in the presence of global pollution externalities, efficiency can be restored by implementing either a uniform global pollution price or a globally linked pollution permit market. But to derive the welfare impact of these policies on different countries, one has to also consider equity, which means specifying a distribution scheme for tax revenues (D'Autume et al., 2016), or an initial distribution pattern for pollution permits (Bretschger, 2017). This especially applies for a policy affecting the economy significantly, as in the case for climate change. The welfare of countries is affected by pollution, but also by the impacts of environmental policy. It can be studied by adopting a macroeconomic setup with pollution externalities where country heterogeneities, such as the differences in pollution intensity, are taken into account. Moreover, as the stock of greenhouse gases and climate policies interact with the growth process in the economy, a dynamic perspective should be adopted. Finally, taking a world planner perspective allows the combination of the efficiency requirement with equity considerations because the planner aggregates over the countries' welfare (Chichilnisky and Heal, 1994). Specifically, the utility functions of the different countries reflect the marginal valuations of consumer goods, which are typically related to income. Thus, a planner solution for a world with dynamic heterogeneous economies characterizes a global optimum, which can serve as a guideline for international environmental policy. This is where the present paper makes a contribution. It goes beyond the economic request to achieve efficiency through a global carbon price, which neglects the impact of this price on a country's welfare.

We derive optimal contributions of the countries to global climate policy in a model of endogenous growth with polluting capital. Countries are heterogeneous with respect to income and pollution intensities; abatement technology is global due to international knowledge diffusion.[1] The agreed temperature targets of international climate treaties involve specific carbon budgets, which set a clear benchmark for our study and define the scope of our analysis. The focus of our paper is on analytically deriving optimal policy contributions of countries, while the explicit derivation of optimal climate targets are a different topic which has been dealt with in literature.[2] Specifically, we consider the case where policy sets a ceiling to pollution stock and distributes pollution permits to the countries. We first use a simple one-country setting to develop the methodology. Then we apply the framework to multiple countries with different kinds of heterogeneities.[3] We adopt a planner perspective to establish the global optimum and then show if and how the optimum can be replicated with a market solution and a specific initial distribution of pollution permits.

The paper distinguishes the two cases with capital mobility and international permit trade and without capital mobility and international permit trade. We believe that the comparison of the solutions for closed and fully open economies is a major contribution because the economic reality is between the two cases. Income differences and the growth pollution trade-off will be essential for the results. We find that more developed countries receive more permits than the less developed, but have to pay higher pollution prices in the case of no capital mobility and no international permit trade. Once we allow for capital movements and free permit trade, more developed countries receive fewer permits than the less developed or even no permits at all.

The results of the paper are important for both climate policy and economic development. In terms of an optimal climate policy design at the global level we show that efforts of the different countries should not be equalized in absolute per capita terms but in terms of marginal utilities. With incomplete international capital mobility and not fully integrated regional permit markets, the most realistic case, international carbon prices will typically not equalize in the optimum. In terms of development we show that an uneven global distribution of physical capital combined with a low aggregate pollution ceiling prevent per capita incomes in the different countries from converging in the long run.

Our paper is related to different strands of literature. An early contribution deriving optimal carbon policies across countries is Chichilnisky and Heal (1994). They model the atmosphere as a public good and find that, for conventional utility functions and abatement provision, the social optimum implies lower levels of abatement in poor countries than in rich countries. They conclude that the requirement of international equalization of marginal abatement cost either ignores distributional issues or assumes unrestricted lump-sum transfers between the countries. D'Autume et al. (2016) show that the world carbon price should be uniform, even in a second-best framework where public goods have to be financed through distortionary taxation and the cost of public funds has to be weighted against the utility of public goods. But this result only holds when lump-sum transfers between countries are possible without restriction. Conversely, if transfers between governments are not possible, international differentiation of the carbon price is the only way to take care of equity concerns. Hillebrand and Hillebrand (2019) develop a dynamic general equilibrium model with an arbitrary number of different regions to study the economic consequences of climate change under alternative climate policies. They show that using especially chosen weights attached to the interests of different countries the optimal solution leads to a Pareto improvement relative to the laissez faire solution. For the sectoral level, Hoel (1996) shows that a carbon tax should not be differentiated between polluting and non-polluting sectors when import and export tariffs are available for all goods. Like these contributions, we adopt an international setup but add the endogenous growth perspective and an analysis of the growth-pollution trade-off with environmental externalities.

By focusing on country contributions to global climate policy the paper is related to the analysis of equity principles in policy by Rose et al. (1998) and Konow (2003) and the applications to environmental economics in Grasso (2007), Page (2008) and Johansson-Stenman and Konow (2010). Specific rules for burden sharing in climate policy based on equity principles are derived

[1] Universal availability prevails for many important new technologies such as solar panels.

[2] To do so, Nordhaus (2017) and Golosov et al. (2014) introduce a negative effect of warming on factor productivity; this could also be implemented in our approach but Bretschger and Pattakou (2019) show that the results critically depend on the form of the applied climate damage functions, which vary very widely in the literature; we thus prefer to restrict the analysis to optimal contributions when carbon budgets are given, which is a novel contribution to the literature.

[3] As we do not focus on climate impacts we abstract from regional heterogeneity with respect to climate damages, which is dealt with in Brock and Xepapadeas (2021).

and discussed in Lange et al. (2007), Mattoo and Subramanian (2010), and Bretschger (2013); egalitarian access to carbon space is proposed by Bode (2004) and BASIC (2011). These policy proposals do not relate to standard welfare theory of economics, while the present contribution embeds the equity topic in a social planner approach based on standard utility theory. We start from first principles and develop a full-fledged dynamic macroeconomic setup to derive optimal solutions for global climate policy.[4]

In order to put optimal solutions into practice, international climate negotiations should provide clear guidelines for effective climate policies worldwide, which is an extremely difficult task. These policies are implemented on the country level and, for an international treaty, all countries must ultimately agree to the joint decisions. One of the main problems is that countries are very different in terms of income and pollution intensity. Applying the general notion of the Pareto principle one could seek for solutions where no single country would lose from an international climate treaty; this approach has been labeled "international Paretianism" (Posner and Weisbach, 2012). Since, at the global level, the benefits of the policy outweigh its costs, such a solution should exist, at least in principle. However, finding a concrete distribution scheme in practice seems to be almost impossible. What is more, there are fundamental issues with a solution which would imply that climate vulnerable and poor countries would have to compensate pollution intensive and richer countries for decarbonization. This seems neither fair nor desirable for the development of less developed countries. So if countries act in a purely selfish and short-sighted way, it is very difficult to reach an effective global climate agreement. But in reality, additional forces like the benefits of coalition formation, positive externalities from technologies and policies, and extrinsic motivation of negotiators also play a role in the negotiation process (Bretschger, 2017). Our paper provides optimal solutions that can guide actual policy making under these conditions.

In terms of instrument choice for climate policy, Weitzman (2014), Stiglitz (2015) and Cramton et al. (2015) favor a world uniform carbon tax which is a clear one-dimensional target, facilitating negotiations and preventing free riding on other countries' efforts. A major problem is the lack of acceptance of the price mechanism among the general public. Moreover, McKenzie and Ohndorf (2012) argue that revenue-raising instruments, such as carbon taxes, are suboptimal because they give rise to unproductive rent seeking. Conversely, a pollution quota and the international distribution of pollution permits avoids these problems. This instrument refers to pollution quantities that are more intuitive for the public and addresses the countries' equity concerns in a direct way, especially with freely allocated pollution permits. We will solve our model formally for the international allocation of pollution permits but could easily reinterpret our results in terms of carbon taxes with international redistribution of tax revenues. We will compare the case of no capital movement and no permit trade with the regime of free capital movement and full permit trade and derive the consequences for the different countries.[5]

The remainder of the paper is organized as follows. Section 2 develops the general setup for the social optimum and the decentralized equilibrium. In Section 3, we introduce multiple heterogeneous countries. Section 4 analyzes the impact of capital and permit trade. In Section 5 we discuss optimal policies. Section 6 concludes.

## 2. General setup

We first develop and solve the model for a single economy which may be thought of as either a single country or the world economy. Applications of the framework to multiple heterogeneous countries are made in subsequent sections, where we distinguish the cases without and with international trade and add the conclusions for optimal policies.

### 2.1. Social optimum

We consider a single-good economy in which output is produced by using capital $k$ with a linear technology (Rebelo, 1991) represented by factor productivity $A > 0$. Capital fully depreciates within one period. Output can be used for consumption or for building future capital stock. Thus, if the capital stock at the beginning of time period $t$ is $k_t$ (and hence the output in period $t$ is $Ak_t$) and the consumption in this period is $c_t$, then the capital stock at the beginning of period $t+1$, $k_{t+1}$, cannot be higher than $Ak_t - c_t$. For clarity, it should be noted that the beginning of time period $t$ coincides with the end of period $t-1$. The initial stock of capital is given at some level $k_0 > 0$.

Capital use is carbon polluting. Its impact on emissions is given by pollution intensity $\nu > 0$ and abatement efficiency, which grows due to exogenous technical progress in abatement at a rate $1/\gamma$, where $0 < \gamma \leq 1$. If at time $t$ the capital stock is $k_t$, the level of emissions is equal to $\gamma^t \nu k_t$, where $\gamma^t$ is the $t$th degree of $\gamma$. We assume that

$$\gamma A > 1.$$

Suppose that at the end of time period $t = 0$, the social planner sets the carbon budget (i.e. emissions aggregated over all time periods) to level $E_0 > 0$ so that the capital stock path $k_t,\ t = 0, 1, \ldots$, satisfies the inequality $\sum_{t=1}^{\infty} \gamma^t \nu k_t \leq E_0$.

The representative consumer maximizes intertemporal utility with a log felicity function and a discount factor $\beta$ which is assumed to be such that

$$\beta A > 1.$$

[4] The dynamic aspects of climate change and climate policy are studied in Bretschger and Valente (2011), where country heterogeneity is introduced, and in Dietz and Venmans (2017), which derives optimal policies in the light of recent advances in climate sciences for the world economy.

[5] This complements the findings of Böhringer et al. (2014) who find that pollution intensive economies generally have conflicting interests with less polluting countries about admitting more countries to a permit trading coalition.

Thus, the social planner solves the program

$$\max \sum_{t=0}^{\infty} \beta^t \ln c_t, \tag{1}$$

$$k_{t+1} + c_t \le Ak_t, \; t = 0, 1, \dots, \tag{2}$$

$$\sum_{t=1}^{\infty} \gamma^t \nu k_t \le E_0, \tag{3}$$

$$k_t \ge 0, \; t = 1, 2, \dots. \tag{4}$$

The Lagrange multipliers associated with the corresponding constraints are denoted as follows: we label the multiplier for the aggregate goods constraint (2) by $p_t$ and interpret it as usual as the shadow price of the aggregate good produced in period $t$, while we label the multiplier for the emissions constraint (3) by $q$ and interpret it as the shadow price of emissions.

We show in Appendix A that if the constraints on emissions are binding, $q > 0$, and hence $\sum_{t=1}^{\infty} \gamma^t \nu k_t = E_0$, then

$$\lim_{t\to\infty} \frac{k_{t+1}}{k_t} = \lim_{t\to\infty} \frac{c_{t+1}}{c_t} = \frac{\beta}{\gamma}. \tag{5}$$

Thus, the long-run growth rate of the economy does not depend on total factor productivity $A$, as could have been expected from an endogenous growth perspective. It is rather determined by the impatience of households ($\beta$) and the development of the abatement technology ($\gamma$), which reveals the dominant impact of the aggregate pollution restriction in this economy. It should be noted that positive long-run growth of the economy is possible if and only if $\beta > \gamma$.

### 2.2. Decentralized equilibrium

In a next step we decentralize the optimal solution of the central planner problem (1)–(4) to a competitive equilibrium. We choose the single good as the numeraire, so that its market price is unity, and denote the price of emissions at the end of period $t$ by $\pi_t$. As usual, we assume that a representative producer represents the production sector and a representative consumer represents households.

In each time period $t$ the representative producer myopically maximizes its profit by solving the following problem

$$\max_{k_t \ge 0} \{Ak_t - (1 + r_t)(k_t + \pi_{t-1}\gamma^t \nu k_t)\}, \tag{6}$$

where $1 + r_t$ is the gross interest rate in period $t$ and $\pi_{t-1}$ is the current value price of emissions at the end of period $t - 1$, which coincides with the beginning of period $t$. Here it is assumed that the representative producer buys the pollution permits necessary for production in period $t$ at the beginning of this period (*ex ante*).[6] Clearly, in equilibrium the profit must be equal to zero.

To enforce pollution restrictions in the decentralized equilibrium, the government allocates a pollution quota in the form of permits to households equal to the amount of $E_0$. Like capital, the permits are individual assets; they are freely tradable on the permit market. Aggregate pollution quantity is fixed like the stock of an exhaustible resource; hence, in an intertemporal setup, pollution permits prices have characteristics similar to exhaustible resource prices, reflecting their increasing scarcity over time.

The total wealth of the representative consumer is equal to the total output[7] $Ak_0$ plus the stock of permits valued at their initial price $\pi_0$, $\pi_0 E_0$.[8] The consumer maximizes her intertemporal utility under the budget constraint that the present value of consumption does not exceed the initial total wealth. Namely, she solves the following problem:

$$\max \sum_{t=0}^{\infty} \beta^t \ln c_t, \tag{7}$$

$$c_0 + \frac{1}{1+r_1} c_1 + \frac{1}{(1+r_2)(1+r_1)} c_2 + \cdots \le Ak_0 + \pi_0 E_0. \tag{8}$$

Since we are interested in the dynamics of our model, it is more illuminating to assume sequential trade and rewrite the single budget constraint as a sequence of budget constraints, complemented by the no-Ponzi-game condition. In this case, the problem the representative consumer solves at time 0 becomes

$$\max \sum_{t=0}^{\infty} \beta^t \ln c_t, \tag{9}$$

$$c_0 + s_0 = Ak_0 + \pi_0 E_0, \tag{10}$$

[6] If we assumed that the representative producer buys the pollution permits necessary for production in period $t$ at the end of this period (*ex post*), then the profit maximization problem would become $\max_{k_t \ge 0}\{Ak_t - [(1 + r_t)k_t + \pi_t \gamma^t \nu k_t]\}$. This problem coincides with problem (6) because, as is shown below (see Proposition 1), the Hotelling rule holds true in our model.

[7] In our model we assume full depreciation. Therefore, the total output implicitly includes non-depreciated capital.

[8] Formally speaking, we assume that the representative producer is owned by the representative consumer who is entitled to receive all the profit. However, the profit is equal to zero in equilibrium because of constant returns to scale. Therefore, it is absent in the budget constraint.

$$c_t + s_t = (1 + r_t)s_{t-1}, \ t = 1, 2, \dots, \tag{11}$$

$$\lim_{T\to\infty} \frac{s_T}{\Pi_{t=1}^T (1 + r_t)} \geq 0. \tag{12}$$

Here, $s_t$ are the savings in period $t$.

A *competitive equilibrium* $(k_t, c_t, E_t, s_t, \pi_t, 1+r_{t+1})_{t=0}^{\infty}$ in the decentralized case is defined by (A) the condition of profit maximization by the representative producer, (B) the condition of utility maximization by the representative consumer and (C) three market clearing conditions for financial, goods and permit markets:

1. The equilibrium in the financial market equilibrium requires that savings are distributed between physical capital and the pollution quotas according to

$$s_t = k_{t+1} + \pi_t E_t, \ t = 0, 1, 2, \dots, \tag{13}$$

where $E_t$ is the carbon budget at the end of period $t$ determined by

$$E_t = E_{t-1} - \gamma^t \nu k_t, \ t = 1, 2, \dots. \tag{14}$$

2. The equilibrium in the goods markets equilibrium requires that

$$c_t + k_{t+1} = Ak_t, \ t = 0, 1, \dots. \tag{15}$$

3. Finally, the equilibrium in the permit market requires that

$$E_t \geq 0, \ t = 0, 1, \dots,$$

or, equivalently, $\lim_{t\to\infty} E_t \geq 0$.

It is noteworthy that, in equilibrium, we also have

$$(1 + r_t)s_{t-1} = Ak_t + \pi_t E_t, \ t = 1, 2, \dots, \tag{16}$$

which follows from (11), (13) and (15), and that the profit of the representative producer is nil and therefore

$$Ak_t = (1 + r_t)(k_t + \pi_{t-1}\gamma^t \nu k_t), \ t = 1, 2, \dots. \tag{17}$$

Since, clearly, $k_t > 0, \ t = 0, 1, \dots$, it follows that the equilibrium interest rate is given by

$$1 + r_t = A \frac{1}{1 + \nu\gamma^t \pi_{t-1}}, \ t = 1, 2, \dots, \tag{18}$$

If the price of pollution permits is zero, then (18) becomes $1 + r_t = A, \ t = 1, 2, \dots$, which is the same as in the AK-model without pollution.

The competitive equilibrium satisfies the following main properties:

(1) In equilibrium, the price of pollution permits satisfies the Hotelling rule known from the theory of exhaustible resources costs (Hotelling, 1931). Namely, the following proposition holds true.

**Proposition 1.** *The price of pollution permits either is zero ($\pi_t = 0, \ t = 0, 1, \dots$), or satisfies*

$$1 + r_t = \frac{\pi_t}{\pi_{t-1}}, \ t = 1, 2, \dots.$$

**Proof.** See Appendix B. □

(2) From Proposition 1 and (18) we obtain that the dynamics of the permits price is given by

$$\pi_t = A \frac{\pi_{t-1}}{1 + \nu\gamma^t \pi_{t-1}}, \ t = 1, 2, \dots, \tag{19}$$

and, therefore,[9] if $\pi_0 > 0$, then

$$\lim_{t\to\infty}(1 + r_t) = \lim_{t\to\infty} \frac{\pi_t}{\pi_{t-1}} = \frac{1}{\gamma}. \tag{20}$$

Thus, *the interest rate converges to the rate of technical progress in the abatement technology*.

(3) Due to the implementation of a permit market, social optimum and competitive equilibrium are essentially the same thing.

[9] Note that (19) can be rewritten as $\gamma^{t+1}\pi_t = \gamma A \frac{\gamma^t \pi_{t-1}}{1+\nu\gamma^t \pi_{t-1}}, \ t = 1, 2, \dots$, and hence if $\pi_0 > 0$, then $\gamma^{t+1}\pi_t \to \frac{\gamma A - 1}{\nu}$ as $t \to \infty$.

**Proposition 2.** *The decentralized equilibrium with free pollution permit trade replicates the social optimum with equilibrium current-value permits prices and the Lagrange multipliers of problem (1)–(4) being associated by*

$$\pi_t = \frac{q}{p_t}, \ t = 0, 1, \dots .$$

**Proof.** See Appendix B. □

(4) The dependence of the permit prices, the total value of permits and the interest rates on the initial carbon budget is monotonic and described in the following proposition.

**Proposition 3.** *For sufficiently low values of $E_0$ (such that the pollution constraint is binding and hence $q$ and $\pi_t$, $t = 0, 1, \dots$, are positive), the equilibrium price of pollution permits, $\pi_t$, and the total value of permits, $\pi_t E_t$, are decreasing and the interest rate $1 + r_t$ is increasing in $E_0$ for each $t = 0, 1, \dots$.*

**Proof.** See Appendix B. □

Next we apply the model setup to the multicountry case to derive optimal policies in a heterogeneous world.

## 3. Many autarkic countries

### 3.1. Social optimum

We consider $n$ different countries and seek for a Pareto optimum, given a global pollution constraint. Neither international permit trade nor capital movement is included in this section, they will be treated separately in the next section. We denote by $\lambda^i > 0$ the Pareto weight of country $i$ ($\sum_{i=1}^n \lambda^i = 1$) in the aggregate welfare. In this paper we assume that $\lambda^i$ reflects country $i$'s population size, but one may also apply additional criteria.[10] In each time period $t$ and in each country $i$, the flow of emissions $e_t^i$ is proportional to the stock of capital in this country, $k_t^i$, with the coefficient of proportionality $\gamma^t v^i$, where $\gamma^t$ is common for all countries and $v^i$ is specific for country $i$:

$$e_t^i = \gamma^t v^i k_t^i, \tag{21}$$

while world emissions in year $t$ are

$$e_t = \sum_{i=1}^n e_t^i.$$

Technology $\gamma^t$ is assumed to be globally available due to international knowledge diffusion. If $\gamma < 1$, then in all countries, the emissions-to-capital ratio decreases over time to ultimately approximate zero, due to technical progress in abatement.

At the end of time period $t = 0$ we set the global carbon budget to some level $E_0 > 0$, i.e. impose the constraint that global emissions aggregated over all time periods cannot exceed $E_0$:

$$\sum_{t=1}^{\infty} e_t \le E_0.$$

Let the initial stock of capital in each country $i = 1, \dots, n$, $k_0^i > 0$ be given. We want to solve the program

$$\max \sum_{i=1}^n \lambda^i \sum_{t=0}^{\infty} \beta^t \ln c_t^i, \tag{22}$$

$$k_{t+1}^i + c_t^i \le A k_t^i, \ i = 1, \dots, n, \ t = 0, 1, \dots \quad (\tilde{p}_t^i), \tag{23}$$

$$\sum_{t=1}^{\infty} \gamma^t \sum_{i=1}^n v^i k_t^i \le E_0, \qquad (\tilde{q}) \tag{24}$$

$$k_t^i \ge 0, \ i = 1, \dots, n, \ t = 1, 2, \dots, \tag{25}$$

where $c_t^i$ is the consumption in country $i$ in period $t$. Compared to the previous section, this program includes $n$ different countries.[11] As in the single-country case, it is possible to prove the existence and uniqueness of an optimal solution of the above problem, to formulate first-order and transversality conditions necessary and sufficient for optimality.

For the comparison of different countries we use exemplary country labels $i$ and $j$. By $E_0$ we denote the carbon budget of country $i = 1, \dots, n$ at the end of period 0. If we know the optimal solution of (22)–(25), then $E_0^i = \sum_{t=1}^{\infty} e_t^i$, $i = 1, \dots, n$.

Solving the problem given in (22)–(25) yields the result summarized in the following proposition.

[10] Konow (2003) and Bretschger (2013) discuss equity principles such as the ability to pay or the merit principle in this context.

[11] Here and below in parentheses we indicate the Lagrange multipliers associated with the corresponding constraints.

**Proposition 4.** *In the social optimum with $n$ different countries and in the absence of capital mobility and international permit trade*

1. *if initially country $i$ pollutes less per capita than country $j$ ($e_0^i/\lambda^i < e_0^j/\lambda^j$), the growth rate of country $i$ and the emissions growth rate will be higher than in country $j$:*

$$\frac{\nu^i}{\lambda^i}k_0^i = \frac{e_0^i}{\lambda^i} < \frac{e_0^j}{\lambda^j} = \frac{\nu^j}{\lambda^j}k_0^j$$
$$\Rightarrow \frac{k_{t+1}^i}{k_t^i} > \frac{k_{t+1}^j}{k_t^j} \text{ and } \frac{e_{t+1}^i}{e_t^i} > \frac{e_{t+1}^j}{e_t^j}, \ i,j = 1,\dots,n, \ t = 0,1,2,\dots; \tag{26}$$

2. *if initially country $i$ pollutes less per capita than country $j$, the optimal amount of pollution permits given to country $i$ is less than the amount given to country $j$:*

$$\frac{\nu^i}{\lambda^i}k_0^i = \frac{e_0^i}{\lambda^i} < \frac{e_0^j}{\lambda^j} = \frac{\nu^j}{\lambda^j}k_0^j \ \Rightarrow \ \frac{E_0^i}{\lambda^i} < \frac{E_0^j}{\lambda^j}, \ i,j = 1,\dots,n. \tag{27}$$

3. *the rate of growth in each country converges to the ratio $\beta/\gamma$:*

$$\lim_{t\to\infty}\frac{k_{t+1}^i}{k_t^i} = \lim_{t\to\infty}\frac{c_{t+1}^i}{c_t^i} = \frac{\beta}{\gamma}, \ i = 1,\dots,n; \tag{28}$$

4. *the ratio of emissions between two countries converge to the ratio of the Pareto weights, irrespective of the countries' pollution intensities:*

$$\lim_{t\to\infty}\frac{e_t^i}{e_t^j} = \frac{\lambda^i}{\lambda^j}, \ i,j = 1,\dots,n, \tag{29}$$

*while the ratio of consumption and the ratio of capital stocks in two countries converge to the ratio of Pareto weights normalized by pollution intensities:*

$$\lim_{t\to\infty}\frac{c_t^i}{c_t^j} = \lim_{t\to\infty}\frac{k_t^i}{k_t^j} = \frac{\lambda^i/\nu^i}{\lambda^j/\nu^j}, \ i,j = 1,\dots,n.$$

**Proof.** See Appendix C. □

Concerning the first and second statements of the proposition, note that initially country $i$ can be less polluting than country $j$ ($e_0^i/\lambda^i < e_0^j/\lambda^j$) either because $i$ is less developed than $j$ ($k_0^i/\lambda^i < k_0^j/\lambda^j$) or because the pollution intensity of $i$ is lower than that of $j$ ($\nu^i < \nu^j$). Although the second statement states that the total amount of pollution permits granted to an initially more polluting country is higher than that granted to a less polluting country, this does not mean that the pollution permits are granted in proportion to the original emissions. It follows from (26) that the ratio of pollution permits to initial emissions is higher in less polluting countries:

$$\frac{e_0^i}{\lambda^i} < \frac{e_0^j}{\lambda^j} \ \Rightarrow \ \frac{E_0^i}{e_0^i} > \frac{E_0^j}{e_0^j}, \ i,j = 1,\dots,n.$$

The second statement of the proposition also implies that efficient contributions to climate policy are unequal between countries, i.e. an optimal distribution of permits partially accommodates the higher demand for permits of more polluting countries to fulfill the policy targets. As we will see shortly it does not mean that optimal pollution prices are internationally equalized.

The third statement of the proposition concerning countries' long-run growth rates looks familiar from the previous section. The fourth one reveals the dynamic adjustment process through capital accumulation. In the long run the emissions in different countries become proportional to their Pareto weights. As for capital stocks and consumption, the picture is more nuanced: if the pollution intensities in two countries are equal, then the capital intensity and per capita consumption converge across countries, while if the pollution intensity in one country is higher, then eventually the capital intensity and per capita consumption in this country become lower.

### 3.2. Decentralization solution

Knowing the optimal values of $e_t^i = \gamma^t \nu^i k_t^i$, at time 0 we allocate to each country $i$

$$E_0^i = \sum_{t=1}^{\infty} e_t^i$$

units of pollution quota and let the *central planner in each country* $i = 1,\dots,n$ solve its optimization problem according to

$$\max \sum_{t=0}^{\infty} \beta^t \ln c_t^i, \tag{30}$$

$$k_{t+1}^i + c_t^i \le Ak_t^i,\ t = 0, 1, \dots, \qquad (\hat{p}_t^i) \tag{31}$$

$$\sum_{t=1}^{\infty} \gamma^t \nu^i k_t^i \le E_0^i, \qquad (\hat{q}^i) \tag{32}$$

$$k_t^i \ge 0,\ t = 1, 2, \dots \tag{33}$$

It is not difficult to verify that

$$\hat{q}^i = \frac{\tilde{q}}{\lambda^i} \text{ and } \hat{p}_t^i = \frac{\tilde{p}_t^i}{\lambda^i},\ i = 1, \dots, n,\ t = 0, 1, \dots .$$

For each country $i = 1, \dots, n$, the solution of problem (30)–(33) can be decentralized as described in Proposition 2. It should be stressed that here we mean decentralization in *each country separately*, which is possible only when the global carbon budget is distributed among the countries.

Then, in country $i$ the price of pollution is given by

$$\pi_t^i = \frac{\hat{q}^i}{\hat{p}_t^i} = \frac{\tilde{q}}{\tilde{p}_t^i},\ t = 0, 1, \dots ,$$

the gross interest rate is

$$1 + r_t^i = A \frac{1}{1 + \nu^i \gamma^t \pi_{t-1}^i},\ t = 1, 2, \dots ,$$

and the dynamics of the price of pollution read

$$\pi_t^i = A \frac{\pi_{t-1}^i}{1 + \nu^i \gamma^t \pi_{t-1}^i},\ t = 0, 1, \dots .$$

Since now the pollution price becomes

$$\pi_t^i = \frac{\tilde{q} c_t^i}{\lambda^i \beta^t},\ i = 1, \dots, n,\ t = 0, 1, \dots ,$$

we have

$$\pi_t^i > \pi_t^j \;\Leftrightarrow\; \frac{c_t^i}{\lambda^i} > \frac{c_t^j}{\lambda^j},\ i, j = 1, \dots, n,\ t = 0, 1, \dots \ .$$

This implies that, if the Pareto weights reflect the size of population, then in a richer country the price of pollution is higher. Also we have

$$1 + r_{t+1}^i > 1 + r_{t+1}^j \;\Leftrightarrow\; \nu^i \pi_t^i < \nu^j \pi_t^j,\ i, j = 1, \dots, n,\ t = 0, 1, \dots . \tag{34}$$

This reflects that the interest rate is lower in a richer country (or in a country with a higher pollution intensity).

These findings have crucial implications for optimal policy contributions: More advanced economies are given an optimal pollution quota such that the resulting permit price is higher than in less developed countries. Permit prices are the most prominent signal for the stringency of environmental policy. Thus, following our global optimality criterion, more developed countries have to make a higher contribution to solving the environmental problem, while less developed countries are allowed to graduate under a less stringent environmental regime. So far, this holds in the absence of capital movement and trade. Indeed, when we allow capital mobility and open the economies for permit trade in the next section, permit prices will equalize. But importantly, there will also be income transferred from rich to poor economies in exchange for the purchase of permits. Whether free permit trade will ultimately be realized on a global level is also a political question: standard economics strongly advocates in favor because of the involved efficiency gains stemming from a decrease in aggregate abatement costs.

In terms of growth, we also see from (34) that

$$\lim_{t \to \infty} \frac{\pi_t^i}{\pi_t^j} = \frac{1/\nu^i}{1/\nu^j} \text{ and } \lim_{t \to \infty} \frac{1 + r_t^i}{1 + r_t^j} = 1,\ i, j = 1, \dots, n.$$

Thus, in the long run, the interest rates across countries converge, even without international capital trade. If the pollution intensities are equal across countries, the pollution prices also converge.

## 4. Capital mobility and international permit trade

### 4.1. Pareto optimum

We now allow for international exchange of capital and pollution permits. International cost competition favors the countries with lowest production cost. Given our linear technology based on endogenous growth theory, cost competition rests on the costs of pollution in our model. Assuming perfect international trade and keeping unequal pollution intensities between the countries the model solution is straightforward: because pollution is costly, all capital and production are moving to the least polluting country, in the social optimum and in the decentralized equilibrium. This is obviously a quite drastic outcome. If we modified our model by assuming partial instead of full depreciation of capital, the relocation of production would not be immediate but happen gradually over time.[12] Introducing such a transition phase would not alter the outcome in the long run. However, when we include international technology transfers to support countries with unfavorable pollution intensities, cleaner technologies can substitute for dirtier ones in each country and international emission coefficients would eventually converge.

There are three main reasons to focus on the outcome of such a convergence process and to assume from now on that the pollution intensities are identical in all countries, i.e. $v^i = v,\ i = 1, \dots, n$. First, we aim to concentrate the analysis on the impact of different income levels for optimal policy contributions by the countries. The central question how to organize an efficient burden sharing in climate policy between wealthy and less developed countries can only be addressed properly when we remove boundary solutions such as the concentration of production in a single country. Second, we aim to analyze international capital trade with inner solutions in a context where countries are at different stages of development; in our one-sector setup, capital transfers can also be interpreted as income transfers. Third, permit markets allow for efficient pricing of pollution but are only instructive for welfare analysis when considered jointly with capital endowments that are positive for more than one country.

First we focus on the role of free capital transfers, which we add to our multicountry setup. Let the initial stock of capital in each country $i = 1, \dots, n$, $\hat{k}^i_0 > 0$ be given. Then, the program we consider becomes

$$\max \sum_{i=1}^{n} \lambda^i \sum_{t=0}^{\infty} \beta^t \ln c^i_t, \tag{35}$$

$$\sum_{i=1}^{n} k^i_{t+1} + \sum_{i=1}^{n} c^i_t \le \sum_{i=1}^{n} A k^i_t,\ t = 0, 1, \dots, \tag{36}$$

$$\sum_{t=1}^{\infty} \sum_{i=1}^{n} \gamma^t v k^i_t \le \sum_{i=1}^{n} E^i_0,\ t = 0, 1, \dots, \tag{37}$$

$$k^i_0 = \hat{k}^i_0,\quad k^i_t \ge 0,\ E^i_0 \ge 0,\ i = 1, \dots, n,\ \sum_{i=1}^{n} E^i_0 = E_0. \tag{38}$$

To describe the solutions to this program, consider the maximization problem (1)–(4) with $k_0 = \sum_{i=1}^n \hat{k}^i_0$. It is easy to show that if $(c_t, k_{t+1})_{t=0}^{\infty}$ is the solution to (1)–(4), then one of the solutions to (35)–(38) is determined as follows:

$$E^i_0 = \lambda^i E_0,\ c^i_t = \lambda^i c_t,\ k^i_t = \lambda^i k_t,\ i = 1, \dots, n,\ t = 0, 1, 2, \dots .$$

What is important here is that the socially optimal proportion of the consumption of country $i$ in the world consumption is equal to its Pareto weight $\lambda^i$. As for the capital stocks and the carbon budgets, it is clear that if, for any $t$, we replace the equalities $k^i_t = \lambda^i k_t$ $i = 1, \dots, n$, by the condition that $\sum_{i=1}^n k^i_t = k_t$, and determine carbon budgets $E^i_0$, $i = 1, \dots, n$, in a corresponding way, we will also obtain a social optimum.

An optimal outcome will be obtained if we redistribute the initial stock of capital, $\sum_{i=1}^n \hat{k}^i_0$, and the initial amount of permits, $E_0$, between countries in proportion to their Pareto weights ($k^i_0 = \lambda^i \sum_{j=1}^n \hat{k}^j_0$ and $E^i_0 = \lambda^i E_0$) and allow consumers in each country $i$ to solve their own program (30)–(33).

### 4.2. Decentralized equilibrium

In this subsection we allow international capital mobility and introduce pollution permits which can be traded freely between the countries. Similar to the analysis in the previous sections we ask whether it is possible to replicate the optimal solution to the program given in (35)–(38) in the decentralized case. Following the last subsection it is straightforward to state that the answer would be yes, provided we could freely distribute the pollution permits and, in addition, redistribute the initial capital stocks. Then, optimality conditions could easily be arranged. But, of course, in the real world it is not possible to redistribute capital stocks, so the plan is not compatible with the concept of the decentralized approach. Also, given our linear $AK$ technology, there is *a priori* no incentive for market participants to transfer capital from rich to poor economies. Thus, the realistic question is whether it is possible to decentralize the optimal solution of (35)–(38) if we can freely distribute the pollution permits but cannot redistribute the initial capital stocks. In terms of climate policy contributions we can then also answer the question how such an efficient allocation of pollution permits would look like.

Suppose we are given a feasible redistribution of the initial world capital stock $(k^i_0)_{i=1}^n$ $(\sum_{i=1}^n k^i_0 = \sum_{i=1}^n \hat{k}^i_0)$ and a feasible distribution of pollution permits, $(E^i_0)_{i=1}^n$ $(\sum_{i=1}^n E^i_0 = E_0)$ with

$$Y^i = A k^i_0 + \pi_0 E^i_0$$

being positive for all $i = 1, \dots, n$. Pollution permits are internationally tradable. We again denote by $1 + r_t$ the (gross) interest rate in period $t$ and by $\pi_t$ the (world) price of pollution in the end of period $t$. The representative consumer in each country $i = 1, \dots, n$ then solves:

$$\max \sum_{t=0}^{\infty} \beta^t \ln c^i_t$$

[12] The gradual adjustment of capital only arises if capital is irreversible, i.e. cannot be transformed back to consumption output. Such a setup is considered in Baldwin et al. (2020).

$$c_0^i + s_0^i = Y^i,$$

$$c_t^i + s_t^i = (1 + r_t)s_{t-1}^i,\ t = 1, 2, \dots .$$

$$\lim_{T\to\infty} \frac{s_T^i}{\Pi_{t=1}^T (1 + r_t)} \geq 0.$$

As for the representative producer in country $i = 1, \dots, n$, at each time $t$ it maximizes its profit by solving the following problem:

$$\max_{k_t^i \geq 0} \{ A k_t^i - (1 + r_t)(k_t^i + \pi_{t-1}\gamma^t \nu k_t^i) \},$$

Again, in equilibrium the profit must be equal to zero.

These programs are similar to the representative consumer and producer problems in Section 2. Note however, that we have now added country labels $i$ for consumption, savings, capital stocks, and emission quantities.

A world competitive equilibrium is again defined by the conditions for the financial, goods and permit markets:

1. In a world competitive equilibrium on the financial market, savings are distributed between physical capital and pollution quotas as follows
$$\sum_{i=1}^n s_t^i = \sum_{i=1}^n k_{t+1}^i + \sum_{i=1}^n \pi_t E_t^i,\ t = 0, 1, \dots ,$$
where $E_t^i$ is the carbon budget of county $i$ at the end of period $t$.
2. A world competitive equilibrium in the goods market now requires that
$$\sum_{i=1}^n c_t^i + \sum_{i=1}^n k_{t+1}^i = \sum_{i=1}^n A k_t^i,\ t = 0, 1, \dots .$$
3. A world competitive equilibrium in the permit market requires that
$$\sum_{i=1}^n E_t^i \geq 0,\ i = 1, \dots, n,\ t = 0, 1, \dots ,$$
where, in contrast to Section 2 we have to consider aggregate emissions on a world level, so that the balance of pollution permits reads
$$\gamma^{t+1}\nu \sum_{i=1}^n k_{t+1}^i + \sum_{i=1}^n E_{t+1}^i = \sum_{i=1}^n E_t^i,\ t = 0, 1, \dots .$$

The return to savings (interest rate), $r_{t+1}$, is given by

$$(1 + r_t)\sum_{i=1}^n s_{t-1}^i = \sum_{i=1}^n A k_t^i + \sum_{i=1}^n \pi_t E_t^i,\quad t = 1, 2, \dots ,$$

and again the Hotelling rule holds true

$$1 + r_t = \frac{\pi_t}{\pi_{t-1}},\ t = 1, 2, \dots .$$

It is easy to check that in equilibrium, just like in the single-country case, the dynamics of the pollution price are given by

$$\pi_t = A\frac{\pi_{t-1}}{1 + \nu\gamma^t \pi_{t-1}},\ t = 1, 2, \dots ,$$

and again in the long run (see (20)) the interest rate converges to the rate of technical progress in the abatement technology:

$$\lim_{t\to\infty}(1 + r_t) = \lim_{t\to\infty} \frac{\pi_t}{\pi_{t-1}} = \frac{1}{\gamma}.$$

It should be stressed that for a given initial world stock of capital, $k_0 = \sum_{i=1}^n \hat{k}_0^i > 0$, and a world emission quota $E_0 > 0$, in a world competitive equilibrium, the equilibrium prices of pollution permits, $\pi_t,\ t = 0, 1, 2, \dots$, and the interest rates, $1 + r_{t+1},\ t = 0, 1, 2, \dots$, are uniquely determined[13] and do not depend on the initial distribution of the capital stock, $(k_0^i)_{i=1}^n$, and the emission quota, $(E_0^i)_{i=1}^n$, among the countries. This property constitutes a modern application and verification of the famous Coase theorem.

It should be highlighted that the exact proportion in which the savings of country $i$ at time $t$, $s_t^i$, are divided between physical capital and pollution quotas is indeterminate (and irrelevant) in equilibrium. Moreover, for each country, it is possible to own foreign capital and pollution permits. At the same time, there is an equilibrium in which all savings of each country take the form of home capital and pollution permits at all times $t$ except time $t = 0$, when the distribution of capital and permits is taken as given.

[13] This follows from the uniqueness of the equilibrium in the single-country case.

However, *it is the initial distribution of capital and permits that determines the welfare of different countries*.

The consumption stream of the representative consumer in country $i$ in equilibrium depends on its initial stock of capital and permits. More specifically,

$$c_0^i = (1-\beta)Y^i,\ c_1^i = (1-\beta)\beta(1+r_1)Y^i,\ ..., c_t^i = (1-\beta)\beta^t(1+r_t)...(1+r_1)Y^i, \dots . \tag{39}$$

It follows from (39) that the utility of the representative consumer in country $i$ is

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t \ln c_t^i = \sum_{t=0}^{\infty} \beta^t \ln(1-\beta) + \sum_{t=1}^{\infty} \beta^t \ln \beta \\
&+ \sum_{t=1}^{\infty} \beta^t \ln(1+r_1) + \sum_{t=2}^{\infty} \beta^t \ln(1+r_2) + \cdots + \sum_{t=0}^{\infty} \beta^t \ln Y^i \\
&= \frac{1}{1-\beta}\ln(1-\beta) + \frac{1}{1-\beta}\ln\beta + \sum_{t=1}^{\infty} \beta^t(1+r_t) + \frac{1}{1-\beta}\ln Y^i,
\end{aligned}$$

and hence the world welfare is

$$\sum_{i=1}^{n} \lambda^i \sum_{t=0}^{\infty} \beta^t \ln c_t^i = \frac{1}{1-\beta}\ln(1-\beta) + \frac{1}{1-\beta}\ln\beta + \sum_{t=1}^{\infty} \beta^t(1+r_t) + \frac{1}{1-\beta}\sum_{i=1}^{n} \lambda^i \ln Y^i.$$

Thus, to maximize the world welfare in equilibrium by means of redistributing the initial world stock of capital and distributing the world emission quota, it is necessary (and sufficient) to solve the following problem (cf. Hillebrand and Hillebrand (2019)):

$$\max \sum_{i=1}^{n} \lambda^i \ln Y^i, \tag{40}$$

$$\sum_{i=1}^{n} Y^i = \sum_{i=1}^{n} A\hat{k}_0^i + \pi_0 E_0. \tag{41}$$

The solution to this problem is given by

$$Y^i = \lambda^i \left( \sum_{i=1}^{n} A\hat{k}_0^i + \pi_0 E_0 \right),\ i = 1, \dots, n. \tag{42}$$

It is easy to check that if the initial redistribution of the world stock of capital and distribution of emission quota are such that (42) is satisfied, the world welfare in equilibrium is equal to the optimal value to the world welfare optimization problem (35)–(38). In this sense the world social optimum can be decentralized. However, *such a decentralization is based on the assumption that we can redistribute the initial world stock of capital, which is not realistic*.

Is it possible to decentralize the world social optimum if redistributing the initial world stock of capital is impossible, but we are free to distribute the world emission quota? The answer to this question is given by the following proposition.

**Proposition 5.** *Suppose that redistributing the initial world stock of capital is impossible.*

*Then in the case where*

$$\lambda^i \left( \sum_{i=1}^{n} A\hat{k}_0^i + \pi_0 E_0 \right) \geq A\hat{k}_0^i,\ i = 1, \dots, n, \tag{43}$$

*there is a distribution of initial permits $E_0$ among the countries such that the world welfare in equilibrium is equal to the optimal value of problem (35)–(38), i.e. the world social optimum can be fully decentralized.*

*Otherwise for any distribution of $E_0$ among the countries the world welfare in equilibrium is lower than the optimal value of problem (35)–(38), i.e. the world social optimum cannot be decentralized.*

**Proof.** When redistributing the initial world stock of capital is impossible, to maximize the world social welfare we should solve the following maximization problem:

$$\begin{aligned}
&\max \sum_{i=1}^{n} \lambda^i \ln Y^i, \\
&\sum_{i=1}^{n} Y^i = \sum_{i=1}^{n} A\hat{k}_0^i + \pi_0 E_0, \\
&Y^i \geq A\hat{k}_0^i,\ i = 1, \dots, n.
\end{aligned}$$

It is clear that if (43) is satisfied, then the solution and optimal value to this problem are the same as the solution optimal value of problem (40)–(41); otherwise its optimal value is lower than that of (40)–(41). □

It should be noted that for a sufficiently large $E_0$ the emission constraint (37) is not binding and hence $\pi_0 = 0$. For smaller values of $E_0$ the emission constraint is binding and $\pi_0 E_0$ is decreasing in $E_0$. It follows that the optimal solution to problem (35)–(38) can be fully decentralized if the initial distribution of physical capital is not too uneven and the world amount of pollution permits is rather small.

To implement the decentralization, it is necessary to give less developed countries more permits in order to obtain a distribution of the world wealth satisfying

$$Ak_0^i + \pi_0 E_0^i = \lambda^i \left( \sum_{i=1}^{n} A\hat{k}_0^i + \pi_0 E_0 \right), \ i = 1, \dots, n.$$

which would equalize *per capita* wealth of all countries. This may be seen as a quite radical requirement but it follows directly from our global social optimum with equal treatment of all people in the world (if we interpret $\lambda^i$ as the share of $i$th country in the world population).

Empirics show that the aggregate capital stock is highly concentrated on a global level. In our model, an optimal distribution of world wealth is impossible if the initial distribution of physical capital is very uneven and/or the initial value of the world emission quota, $\pi_0 E_0$, is small. In this case all permits will be given to less developed countries, while the most developed countries will not receive any allowances. This conclusion is similar to the notion of an "egalitarian access to carbon space" but is derived from a dynamic economic model as an efficient policy. We summarize and further characterize our findings in the next section.

Finally, note that when the world social optimum can be decentralized, the optimal equilibrium is characterized by full *per capita* consumption equality among the countries (again if $\lambda^i$ is the share of $i$th country in the world population). Otherwise, *per capita* consumption in equilibrium will be unequal forever because, as follows from (39), in a world competitive equilibrium, for any two countries $i$ and $j$,

$$\frac{c_0^i}{c_0^j} = \frac{c_1^i}{c_1^j} = \cdots = \frac{Y^i}{Y^j}.$$

If full *per capita* consumption equality among the countries is unattainable because of impossibility to reallocate the initial capital stocks, a natural question arises: what conditions lead to lower inequality among countries? An answer to this question is suggested by Proposition 3. That proposition implies that when the world carbon budget becomes more stringent, the total value of the world permits becomes higher and hence the space for redistribution of the initial wealth enhances, which leads to higher equality among countries in equilibrium.

## 5. Optimal policies

We are now ready to discuss our main results in the light of the starting point, the efficient and equitable contribution of countries to international climate policy and their impacts. We will distinguish the different cases treated in the paper.

### 5.1. Optimal permit distribution

When global climate policy is based on permit markets, the allocation of pollution permits to countries is a central issue. We have found that if there is no international capital movement and permits are not traded internationally, it is optimal on a global to give *ceteris paribus* more permits to more developed countries, see (27). If, however, capital moves freely and international permit trade becomes possible, the situation is just the opposite: more developed countries receive fewer permits in an optimal distribution. It is then optimal from a global perspective that these countries acquire additional permits via the international market.

We conclude that the decision on an optimal international distribution of permits depends on a question of institutional arrangement, which is whether national permit markets can be linked on a global level or not. This is a highly political issue. Economists would in general favor such a linking for efficiency reasons, but from a political perspective there might be reservations because countries then become interdependent in a crucial policy area.

### 5.2. Permit prices

The prices of pollution permits are the main signal for the stringency of climate policy in a country. We find that in the absence of capital mobility and international permit trade, the equilibrium prices of pollution permits are higher in the rich countries than in the poor countries. This reflects the intense scarcity of pollution rights in developed regions which turns out to be optimal for policy burden sharing on a global level. Compared to the proposal of a uniform world carbon price, where countries keep their tax revenues, we see that developed countries are requested to pay more, given the global optimization.

Of course, as soon as capital moves freely and permits become tradable at the international level, pollution permit prices immediately equalize. This is in the mutual interest of buyers and sellers of permits; a standard result of environmental economics, which is equivalent to the proposal to establish uniform international carbon prices. But the decisive result here is that the optimal allocation of permits to richer countries is such that they induce an income transfer from the rich to the poorer countries with permit trade. Hence we have established that it is optimal to allocate a relatively higher burden of climate policy to the richer countries, provided we take a global welfare perspective as adopted in this paper.

### 5.3. Income convergence

If there is no international permit trade, countries' income levels converge, even if we observe no international capital movement. Pollution restrictions are strong enough to bring about convergence, which is a remarkable result.

If capital movement is allowed and permits are traded internationally, two scenarios are possible:

- If the initial distribution of physical capital is not too uneven and the world amount of pollution permits is small and hence the value of world permits is high, then the distribution of the permits such that all countries are in identical income positions from the first period on is possible, which is a stable condition over time.
- If the initial distribution of physical capital is uneven and the world amount of pollution permits is significant and hence the value of world permits is small, then complete equality between countries is not reached through the distribution of permits and moreover, the countries do not converge in the long run. This happens even when markets are fully globalized.

It is realistic to assume that the world economy is characterized by an uneven distribution of physical capital, but that international climate policy prescribes an aggregate amount of pollution permits which is quite small. Hence, the question of income convergence cannot be answered unambiguously. Global pollution restrictions entail convergence forces, but whether incomes ultimately converge depends on the stringency of the implemented environmental policy.

## 6. Conclusions

Using a multicountry endogenous growth model, we have derived optimal country contributions to international climate policies, which we defined as a cap on global pollution stock. We have found that an optimal policy design typically deviates from identical policy efforts of all the countries. In the adopted world planner approach, efforts are not equalized in absolute terms but in terms of marginal utilities. When capital does not move across national borders and permits are not traded internationally it means that more developed countries have to pay higher pollution prices despite the fact that they receive more pollution permits as an initial endowment. With free capital movement and international permit trade, pollution prices become uniform, more developed countries receive fewer permits in the beginning and marginal abatement costs are equalized internationally.

Our planner approach provides a theoretical guideline for optimal global policies. The international climate negotiations have the difficult task of inducing implementation of such policies in practice. If not in a precise manner and not all at once, the policy steps should at least point in the right direction i.e. move the economies from today's suboptimal state towards a global optimum. In the current climate policy process, instrument choice is delegated to the country level, where not only taxes and permits but also bans and other legal instruments play an important role. All these measures are especially effective when they induce further technical progress in abatement, which would be a possible extension of our approach. Also, the effects of the introduction of a second type of capital which is clean would be interesting to study. This is left for further research.

## Appendix A

In this appendix we prove Eq. (5). The Lagrangian for problem (1)–(4) is

$$\mathcal{L} = \sum_{t=0}^{\infty}(\beta^t \ln c_t + p_t(Ak_t - c_t - k_{t+1})) + q(E_0 - \sum_{t=1}^{\infty}\gamma^t \nu k_t).$$

It is clear that in an optimal solution, the capital stock and consumption are positive at every time ($k_t > 0,\ c_t > 0,\ t = 0, 1, \dots$). Therefore, the first-order conditions for problem (1)–(4) are:

$$p_t = \frac{\beta^t}{c_t},\ t = 0, 1, \dots, \tag{A.1}$$

$$Ap_t = p_{t-1} + \gamma^t \nu q,\ t = 1, 2, \dots, . \tag{A.2}$$

$$k_{t+1} + c_t = Ak_t,\ t = 0, 1, \dots, \tag{A.3}$$

$$\sum_{t=1}^{\infty}\gamma^t \nu k_t \le E_0, \tag{A.4}$$

$$q(E_0 - \sum_{t=1}^{\infty}\gamma^t \nu k_t) = 0, \tag{A.5}$$

and the transversality condition is

$$\lim_{t\to\infty} p_t k_{t+1} = 0. \tag{A.6}$$

From (A.2) we have

$$p_t = \frac{1}{A}p_{t-1} + \gamma^t \frac{\nu q}{A},\ t = 0, 1, \dots .$$

Taking into account (A.1), we get for all $t = 0, 1, \ldots,$

$$\frac{\beta^t}{c_t} = \frac{1}{A}\frac{\beta^{t-1}}{c_{t-1}} + \gamma^t \frac{\nu q}{A}$$

and hence

$$\frac{\beta^t}{\gamma^t}\frac{1}{c_t} = \frac{1}{\gamma A}\frac{\beta^{t-1}}{\gamma^{t-1}}\frac{1}{c_{t-1}} + \frac{\nu q}{A}.$$

It follows that if $q > 0$, then

$$\frac{\beta^t}{\gamma^t}\frac{1}{c_t} \xrightarrow[t\to\infty]{} \frac{\nu\gamma q}{\gamma A - 1}$$

and, therefore,

$$\frac{c_{t+1}}{c_t} \xrightarrow[t\to\infty]{} \frac{\beta}{\gamma}. \tag{A.7}$$

Let us now show that

$$\frac{k_{t+1}}{c_t} \xrightarrow[t\to\infty]{} \frac{\beta}{\gamma A - \beta} \tag{A.8}$$

Indeed, we have

$$\frac{k_{t+1}}{c_t} = \frac{Ak_t}{c_t} - 1 = A\frac{c_{t-1}}{c_t}\frac{k_t}{c_{t-1}} - 1, \; t = 1, 2, \ldots.$$

We know that

$$\frac{c_{t-1}}{c_t} \xrightarrow[t\to\infty]{} \frac{\gamma}{\beta}$$

Since $\gamma A/\beta > 1$, there are three possible scenarios: (1) at some time $k_{t+1}/c_t$ becomes negative; (2) $k_{t+1}/c_t$ converges to $\frac{\beta}{\gamma A-\beta}$; (3) $k_{t+1}/c_t$ goes to infinity. The first scenario is impossible. The third one is also impossible because if $k_{t+1}/c_t$ goes to infinity, then $k_{t+1}/k_t$ converges to $A$ and hence $\sum_{t=1}^{\infty}\gamma^t \nu k_t$ becomes infinitely large, which is impossible. Thus, only the second scenario is possible. This proves (A.8).

Combining (A.7) and (A.8) we obtain

$$\frac{k_{t+1}}{k_t} \xrightarrow[t\to\infty]{} \frac{\beta}{\gamma},$$

which completes the proof of (5).

## Appendix B

In this appendix we prove the Hotelling rule (Proposition 1) and the result that the total value of permits decreases when the quantity of permits increases (Proposition 3).

The detailed proofs of the existence of an optimal solution to problem (1)–(4) and the result that conditions (A.1)–(A.6) are necessary and sufficient conditions of optimality are available from the authors upon request. The same applies for the existence and uniqueness of a competitive equilibrium for the single-economy model and the result that optimum and equilibrium are the same in the single-country model (Proposition 2).

**Proof of Proposition 1.** Substituting (17) into (16) and taking account of (13) and (14) we get for $t = 1, 2, \ldots,$

$$\begin{aligned}(1+r_t)s_{t-1} &= (1+r_t)(k_t + \pi_{t-1}\gamma^t \nu k_t) + \pi_t E_t \\ &= (1+r_t)(k_t + \pi_{t-1}E_{t-1} - \pi_{t-1}E_t) + \pi_t E_t \\ &= (1+r_t)s_{t-1} - (1+r_t)\pi_{t-1}E_t + \pi_t E_t.\end{aligned}$$

Therefore,

$$(1+r_t)\pi_{t-1}E_t = \pi_t E_t, \; t = 1, 2, \ldots,$$

which proves the proposition. □

Now observe that the solution of problem (9)–(12) is given by

$$\begin{cases} c_0 = (1-\beta)(Ak_0 + \pi_0 E_0), \; s_0 = \beta(Ak_0 + \pi_0 E_0), \\ c_t = (1-\beta)(1+r_t)s_{t-1}, \; s_t = \beta(1+r_t)s_{t-1}, \qquad t = 1, 2, \ldots. \end{cases} \tag{B.1}$$

Therefore, in equilibrium

$$k_{t+1} = Ak_t - c_t = \beta A k_t - (1-\beta)\pi_t E_t, t = 0, 1, \ldots. \tag{B.2}$$

**Proof of Proposition 3.**

**Claim 1.** *For $E_0$ such that $\pi_0 > 0$, if $E_0$ increases, then $\pi_t$ decreases for all $t = 0, 1, 2, \dots$.*

**Proof.** Suppose that $E_0$ grows, but $\pi_0$ does not decrease and by (19), $\pi_t$ does not decrease for all $t = 0, 1, 2, \dots$.. Taking account of (B.2) we obtain that $k_1$ decreases and hence $E_1 = E_0 - \gamma \nu k_1$ increases. Repeating the argument we obtain that $k_t$ decreases for all $t = 1, 2, \dots$.. Therefore $\sum_{t=1}^{\infty} \gamma^t \nu k_t$ becomes strictly less than $E_0$ and $\pi_0$ becomes equal to zero, which is impossible. This prove that $\pi_0$ decreases. Since the sequence $(\pi_t)_{t=0}^{\infty}$ be determined by (19), $\pi_t$ decreases for all $t = 0, 1, 2, \dots$. □

**Claim 2.** *For $E_0$ such that $\pi_0 > 0$, if $E_0$ increases, then $1 + r_{t+1}$ also increases for all $t = 0, 1, 2, \dots$.*

**Proof.** It is sufficient to note that by the first claim and (18), $1 + r_{t+1}$ is decreasing in $\pi_t$. □

**Claim 3.** *For $E_0$ such that $\pi_0 > 0$, if $E_0$ increases, then $\pi_t E_t$ decreases for all $t = 0, 1, 2, \dots$.*

**Proof.** Suppose that $E_0$ grows, but $\pi_0 E_0$ does not decrease. Then $c_0$ does not decrease. Therefore, by (B.1), $k_1 = Ak_0 - c_0$ and hence $Ak_1$ do not increase. At the same time, (B.1) implies that $s_0$ does not decrease. Note also that $1 + r_1$ does not decrease by Claim 2. It follows that $Ak_1 + \pi_1 E_1 = (1 + r_1)s_0$ does not decrease. Thus, $\pi_1 E_1$ does not decrease.

Repeating the argument we obtain that $k_t$ does not increase for all $t = 0, 1, 2, \dots$. It follows that $\sum_{t=1}^{\infty} \gamma^t \nu k_t$ does not increase and hence becomes strictly less than $E_0$. This implies that $\pi_0$ becomes zero, which is impossible. □

This completes the proof of Proposition 3. □

## Appendix C

In this appendix we prove Proposition 4.

**Proof of Proposition 4.** It is clear that in an optimal solution, in each country, the capital stock and consumption are positive at every time ($k_t^i > 0,\ c_t^i > 0,\ i = 1, \dots, n,\ t = 0, 1, \dots$). Therefore, the first-order conditions for problem (22)–(25) are:

$$\tilde{p}_t^i = \lambda^i \frac{\beta^t}{c_t^i},\ i = 1, \dots, n,\ t = 0, 1, \dots,$$
$$A\tilde{p}_t^i = \tilde{p}_{t-1}^i + \gamma^t \nu^i \tilde{q},\ i = 1, \dots, n,\ t = 0, 1, \dots.$$

$$k_{t+1}^i + c_t^i = Ak_t^i,\ i = 1, \dots, n,\ t = 0, 1, \dots, \tag{C.1}$$

$$\sum_{t=1}^{\infty} \gamma^t \sum_{i=1}^{n} \nu^i k_t^i \le E_0,$$
$$(E_0 - \sum_{t=1}^{\infty} \gamma^t \sum_{i=1}^{n} \nu^i k_t^i) = 0,$$

and the transversality condition is

$$\lim_{t\to\infty} \tilde{p}_t \sum_{i=1}^{n} k_{t+1}^i = 0.$$

A slight modification of the arguments used in Appendix A shows that these conditions are necessary and sufficient conditions of optimality.

It follows from the first-order conditions that, in optimum, for all $i = 1, \dots, n$ and $t = 1, 2, \dots$, we have

$$\frac{\lambda^i}{\nu^i} \frac{1}{c_t^i} = \frac{1}{\beta A} \frac{\lambda^i}{\nu^i} \frac{1}{c_{t-1}^i} + \frac{\gamma^t}{\beta^t} \frac{\tilde{q}}{A} \tag{C.2}$$

and, hence,

$$\frac{\lambda^i}{\nu^i} \frac{\beta^t}{\gamma^t} \frac{1}{c_t^i} = \frac{1}{\gamma A} \frac{\lambda^i}{\nu^i} \frac{\beta^{t-1}}{\gamma^{t-1}} \frac{1}{c_{t-1}^i} + \frac{\tilde{q}}{A}.$$

It follows that

$$\frac{\lambda^i}{\nu^i} \frac{\beta^t}{\gamma^t} \frac{1}{c_t^i} \xrightarrow[t\to\infty]{} \frac{\gamma \tilde{q}}{\gamma A - 1}$$

and, therefore,

$$\lim_{t\to\infty} \frac{c_t^i}{c_t^j} = \frac{\lambda^i / \nu^i}{\lambda^j / \nu^j},\ i, j = 1, \dots, n.$$

and

$$\lim_{t\to\infty} \frac{c^i_{t+1}}{c^i_t} = \frac{\beta}{\gamma}, \ i = 1, \dots, n. \tag{C.3}$$

By the same argument as in Appendix A,

$$\lim_{t\to\infty} \frac{k^i_{t+1}}{k^i_t} = \frac{\beta}{\gamma}, \ i = 1, \dots, n.$$

Thus, we have proved (28) and (29).

It follows from (C.2) that

$$\frac{v^i}{\lambda^i} c^i_t < \frac{v^j}{\lambda^j} c^j_t \ \Leftrightarrow \ \frac{v^i}{\lambda^i} c^i_{t+1} < \frac{v^j}{\lambda^j} c^j_{t+1}, \ i, j = 1, \dots, n, \ t = 0, 1, 2, \dots. \tag{C.4}$$

At the same time, (C.1), (C.3) and the inequality $\gamma A > 1$, imply

$$c^i_t + \frac{1}{A} c^i_{t+1} + \frac{1}{A^2} c^i_{t+2} + \cdots = A k^i_t, \ i = 1, \dots, n, \ t = 0, 1, 2, \dots. \tag{C.5}$$

and hence

$$\frac{v^i}{\lambda^i} c^i_t + \frac{1}{A} \frac{v^i}{\lambda^i} c^i_{t+1} + \frac{1}{A^2} \frac{v^i}{\lambda^i} c^i_{t+2} + \cdots = \frac{v^i}{\lambda^i} A k^i_t, \ i = 1, \dots, n, \ t = 0, 1, 2, \dots. \tag{C.6}$$

From (C.4) and (C.6) we obtain

$$\frac{v^i}{\lambda^i} k^i_0 < \frac{v^j}{\lambda^j} k^j_0 \ \Leftrightarrow \ \frac{v^i}{\lambda^i} k^i_t < \frac{v^j}{\lambda^j} k^j_t \ \Leftrightarrow \ \frac{v^i}{\lambda^i} c^i_t < \frac{v^j}{\lambda^j} c^j_t, \ i, j = 1, \dots, n, \ t = 0, 1, \dots. \tag{C.7}$$

Taking account of (21), we get

$$\frac{v^i}{\lambda^i} k^i_0 < \frac{v^j}{\lambda^j} k^j_0 \ \Leftrightarrow \ \frac{e^i_t}{\lambda^i} < \frac{e^j_t}{\lambda^j}, \ i, j = 1, \dots, n, \ t = 0, 1, \dots \ .$$

Since $\sum_{i=1}^n E^i_0 = E_0$, this proves (27).

Moreover, from (C.2) we have

$$\frac{v^i}{\lambda^i} c^i_t < \frac{v^j}{\lambda^j} c^j_t \ \Rightarrow \ \frac{c^i_{t+1}}{c^i_t} > \frac{c^j_{t+1}}{c^j_t}, \ i, j = 1, \dots, n, \ t = 0, 1, \dots.$$

Also we can rewrite (C.5) as

$$1 + \frac{1}{A} \frac{c^i_{t+1}}{c^i_t} + \frac{1}{A^2} \frac{c^i_{t+2}}{c^i_t} + \cdots = \frac{A k^i_t}{c^i_t}, \ t = 0, 1, \dots,$$

Therefore

$$\frac{v^i}{\lambda^i} c^i_t < \frac{v^j}{\lambda^j} c^j_t \ \Rightarrow \ \frac{A k^i_t}{c^i_t} = 1 + \frac{1}{A} \frac{c^i_{t+1}}{c^i_t} + \frac{1}{A^2} \frac{c^i_{t+2}}{c^i_t} + \cdots$$

$$> 1 + \frac{1}{A} \frac{c^j_{t+1}}{c^j_t} + \frac{1}{A^2} \frac{c^j_{t+2}}{c^j_t} + \cdots = \frac{A k^j_t}{c^j_t}, \ i, j = 1, \dots, n, \ t = 0, 1, \dots.$$

Since

$$k^i_{t+1} = A k^i_t - c^i_t, \ i = 1, \dots, n, \ t = 0, 1, \dots,$$

we obtain

$$\frac{v^i}{\lambda^i} c^i_t < \frac{v^j}{\lambda^j} c^j_t \ \Rightarrow \ \frac{k^i_{t+1}}{A k^i_t} > \frac{k^j_{t+1}}{A k^j_t}, \ i, j = 1, \dots, n, \ t = 0, 1, \dots.$$

Taking into account (C.7), we get (26) in the main text. □

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